Algorithms 1: The beginning

Algorithms

- The problem
- The solution
- Math
- History

The problem

- There are times when the world is one way, and you would like it to be another
- For example, I am hungry and would like a brownie, but there are no brownies in the house
- In this case, I can go online and find a recipe for brownies and follow it
- Now there are brownies
- Or if I need to get somewhere
- I can go online and get directions on how to get there
- If I follow the directions I will end up at my destination

The solution

- The useful things about recipes and directions is:
- They are easy to follow
  - Anybody with some minimal amount of knowledge and ability can do it
- They produce the desired result
  - If you follow the recipe/directions correctly, you will get what you want
- This is the basis of what an algorithm is

  An algorithm is a set of rules that precisely defines a sequence of operations
- Recipes and directions are both types of algorithms
- Given the importance of computers, the term algorithm is often conflated with program
  - Programs are a subset of algorithms, not all of them

The solution

- Recipes and directions look different
- Here is a recipe

Gooey Fudge Brownie

Ingredients

Portions: 9

- 1/2 cup melted butter, salted
- 1/2 cup cocoa powder
- 1 cup sugar
- 1 Tbsp vegetable oil
- 2 large eggs
- 1 tsp vanilla
- 1/2 cup flour

Directions

1. Preheat oven to 350°F. Grease an 8x8 square pan
2. In a medium bowl combine melted butter and cocoa and sugar stir until fully dissolved.
3. Add eggs one at a time then vanilla and stir until well combined. Stir in flour and salt until the flour is fully combine. Do not overmix.
4. Spread in pan and bake for approximately 20-22 minutes or until the center is slightly set. Be careful not to over-bake! Cool completely then cut into 9 large squares

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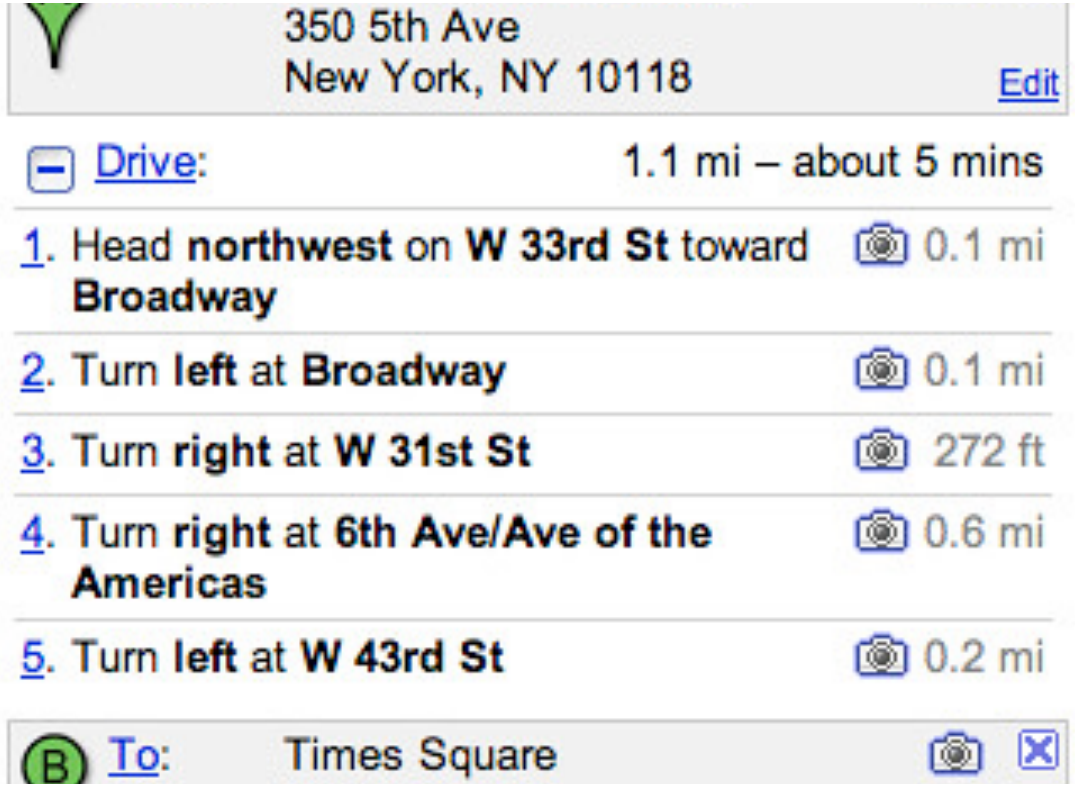
The solution

- And here are directions for traveling from one place to another



The solution

- Despite the differences, there are a few similarities between them
- There are prerequisites
  - Things that must be true before starting the algorithm
- There are steps, and they have an order
  - First this, then that, and so on
- There is some way to know when you are done

The solution

- Consider the brownie recipe
- Prerequisites
  - The ingredients listed, in the amounts stated
  - An oven that gets hot enough for long enough
- Steps
  - The Directions section lays them out in order
- Done?
  - After 20-22 minutes at 350 degrees Fahrenheit
  - And they have cooled off
- The same sort of thing is true for the travel directions

Math

- Pretty much every field has algorithms that describe how to do things within the field
  - The form of the algorithms may differ, but the basics are the same
- Math is no different
- Many things that we take for granted today were new algorithms at some time
  - Multiplying and factoring numbers
  - Finding square roots and testing for prime numbers
- At first mathematicians were fairly casual about what an algorithm actually was
  - If you could figure out how to do it, and the answers were correct, that was good enough
- For the most part algorithms were considered just one of the tools that mathematicians had when solving problems
  - In addition to logic and various types of proofs

Math

- After a while questions began to emerge about this informal use of algorithms
- Mathematicians wanted to know:

✓What are the minimum prerequisites needed for an algorithm?

✓What are the simplest steps that can be used to describe an algorithm?

✓How can we be sure if an algorithm will ever be done?

✓How can we be sure that an algorithm gets the right answer?

- In addition to purely theoretical curiosity, mathematicians were motivated by new machines that could do complicated things like automatically calculate logarithms
- But more discussion means we need to look at the history of algorithms

History

- Before we start, my apologies to anyone I leave out of this lecture
- The point is to provide a very cursory view of how we got to the place we are in
- Not to provide an exhaustive listing of every mathematician and idea that has ever existed
  - Such a book would be very large and contentious
- As a result, I will be leaving out areas that contributed important mathematics in the interest of staying focused and finishing in a reasonable time

History

- Algorithms have been used in many fields since time immemorial
  - We will stick with math
- The earliest algorithms were devised by the usual suspects: the Egyptians, Babylonians, and Indians
- They covered some basic operations
  - Addition, subtraction, multiplication, division
- And some more advanced ones
  - Factorization, finding square roots
- Tools were invented to help
  - The abacus, in particular

History

- Next up were the Greeks
- Euclid's algorithm
  - For finding the greatest common divisor of two positive integers
- The Sieve of Eratosthenes
  - For finding prime numbers
- Euclid also invented the idea of an axiomatic proof, which has deep ties to algorithms
- The tools used included
  - Paper and pen
  - Diagrams of geometric objects
- Eventually the Romans took over
  - They were more interested in practical engineering problems

History

- The Indians and Arabs kept pushing
- The Indians
  - Negative numbers, zero, and infinity
  - The decimal number system
  - Solution for quadratic equations
  - Combinatorics
- The Arabs
  - Algebra (the study of variables in equations and their manipulation)
  - Induction
  - Irrational numbers
- The tools remained mostly the same, but progress was made in terms of standardized structures for stating and solving problems

History

- By inventing negative numbers, it is not meant that they were unknown before Brahmagupta in the 7th century
  - They are attested in China in around 200 BCE
- He gets credit because he laid out rules for manipulating them: adding, subtracting, multiplying, etc.
- In other words, it takes a combination of the concept (e.g., negative numbers) and algorithms for a mathematical idea to be useful

History

- After the Greeks the Romans (and the rest of the Western world) did not do much with math
  - There was some work in logic, which became useful later
- This was at least partially due to the fact that Roman numerals are hard to work with
  - That is, the algorithms for adding, subtracting, etc., Roman numerals are complicated
- This changed when a couple of Arabic books were introduced to the Western world in the early 1200s

History

- Around 825, Muhammad ibn Musa al-Khwarizmi wrote (among others):
- kitāb al-ḥisāb al-hindī
  - Latin: Algoritmi De Numero Indorum
  - "Book of Indian computation"
- kitab al-jam' wa'l-tafriq al-ḥisāb al-hindī
  - "Addition and subtraction in Indian arithmetic"
- al-Kitāb al-Mukhtaṣar fī Ḥisāb al-Jabr wal-Muqābalah
  - Latin: “Liber Algebræ et Almucabola”
  - “The Compendious Book on Calculation by Completion and Balancing”
- In the early 12th century Latin translations of these books started to show up in Europe
- Math (and algorithms) was never the same

History

- These books introduced to Europe:
- Arithmetic
  - Decimal numbers and notation
- Algebra
  - From the Arabic al-jabr (“the reunion of broken parts”)
  - Not just the word, but much of the actual practice
- The term algorithm
  - From the Latinization of Al-Khwarizmi into Algoritmi (or alghoarismi or algorismi)
  - Algorism was the word used to describe following the rules for manipulating Indian numerals as laid out by Al-Khwarizmi

History

- The books were also a boost in a few other ways
- ✓It helped Europe catch up to more mathematically advanced areas
- ✓It loosened the grip of geometry on European math
- ✓It illustrated the importance of being able to carry out sequences of operations in order to discover mathematical facts
  - As opposed to, say, writing a proof
- ✓It showed how important a good notation could be
- At the same time some new tools were being invented
  - In particular the astrolabe (for calculating position on the globe)
  - And other practical tools for measuring and calculating

History

- The Europeans took the ball and ran with it
  - I am not sure if soccer was invented yet
- After a while the Europeans discovered something: actually doing the calculations was boring
- This only got worse as practical applications for math increased
  - Navigation, physics, engineering, warfare, …
- For various reasons Europeans had been interested in clocks and other mechanical devices
- And somebody got the idea that maybe the mechanical device could do all the calculating while people did the interesting work

History

- The history of mechanical calculation is too big for this lecture
- It does lead to an important question, however
- How can we describe the algorithm for a calculation in such a way that the machine can perform it?
  - So a person does not have to