

Propositions

A proposition is a formal statement that is true or false (but not a matter of opinion).

- Examples of propositions

- 37 is a prime number True
- a mouse is a small cat False

- Not a proposition:

- $x = 2$

- Building blocks of propositions: *primitive* or *atomic* propositions.

- Notation: p, q, r, \dots

Negation

- We can negate a proposition:
'37 is an odd number' \rightarrow 37 is NOT an odd number
- The truth value is opposite: the negation of a true proposition is false, the negation of a false proposition is true.
- Notation: $p \rightarrow$ negation $\neg p$ not p $\sim p$ \bar{p}
- Double negation: $\neg(\neg p) = p$

Conjunction (and)

- **and** joins two propositions together.
- Notation: $p \wedge q$ p and q
- p **and** q is true if both p and q are true

Conjunction - example

- 5 is a prime number **and** 7 is an even number.

p
 T

q
 F

$p \wedge q$ False

- 5 is a prime number **and** 7 is an odd number.

p
 T

$\neg q$
 T

$p \wedge \neg q$ T

Disjunction (or)

- **or** joins two propositions together.

- Notation: $p \vee q$

- p **or** q is true if
 - p is true
 - q is true
 - p and q are true

Disjunction

- 5 is a prime number **or** 7 is an even number. T
- 5 is a prime number **or** 7 is an odd number. T
- 9 is a prime number **or** 7 is an even number. F

Brackets

When joining many propositions, you need brackets to disambiguate.

Example:

- p : 8 is a prime number. F
- q : 9 is divisible by 4. F
- r : $1 + 1 = 2$ T

■ $(p \wedge q) \vee r$ True

(Handwritten annotations: A red bracket under $p \wedge q$ is labeled 'F'. A green bracket under r is labeled 'T'. The word 'True' is written in green to the right.)

■ $p \wedge (q \vee r)$ False

(Handwritten annotations: A red 'F' is written below p . A green bracket under $q \vee r$ is labeled 'T'. The word 'False' is written in red to the right.)