Overview of the course

- Propositional logic lecture 1
- Proof techniques lectures 2-4
- Set theory today and next lecture
- Relations lectures 7-8
- Functions lectures 8-9
- Combinatorics lectures 10-11

Book: Chapter 2, sections 2.1, 2.2, 2.3

Sets

- A set: collection of specified objects

$A = \{1, 2, 3, 4\}$ ↳ element (member) $\quad 1 \in A$

$\mathbb{N}, \mathbb{Z}, \mathbb{R}$

- Sets are not ordered $\quad \{1, 2, 3, 4\} = \{2, 4, 3, 1\}$

- Sets have no duplicate elements $\quad \{\cancel{1}, 1, 2\} = \{1, 2\}$

- Sets can be finite or infinite

- Can be empty $\quad \{\}, \phi$

- Cardinality of a set = number of elements

$|\{1, 2, 3, 4\}| = 4, \quad |\{\}| = 0, \quad |\{1, \{2, 3\}\}| = 2$

Describing sets

$$A = \{1, 2, 3, 4\}$$

$$A = \{x : x \in \mathbb{N} \land 1 \leq x \leq 4\}$$

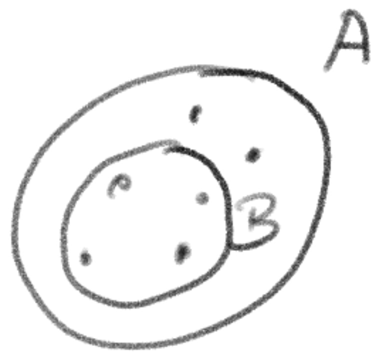
$$A = \{x \in \mathbb{N} : x < 5\}$$

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$$

Subsets

The set B is a subset of the set A $\overset{DEF}{\Leftrightarrow}$ every element of B is also an element of A



B = { x : x is a student at DACS }

A = { x : x is a student at UM }

↳ all members of B are also in A

$B \subseteq A$

↳ is a subset.

$$B \subseteq A \Leftrightarrow \forall x \in B : x \in A$$

$$\forall x : (x \in B) \rightarrow (x \in A)$$

$\emptyset \subseteq A$ ✓

$A \subseteq A$ ✓

$$\{\} \neq \{\emptyset\}$$

Set equality

$A = B$

$A = B$ if they have the same elements

$\{2,3\} = \{x : x^2 - 5x + 6 = 0\}$

A B

$\{-1,-2,1,2\} = \{x : x^2 = 4 \lor x^2 = 1\}$

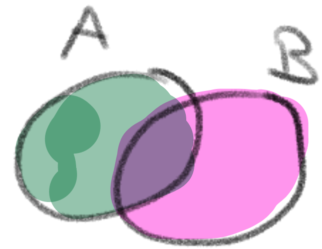
All elements of A are elements of B and all element of B are elements of A

$(A \subseteq B \land B \subseteq A) \iff A = B$

Union, intersection, difference of sets

- union $A \cup B$

  the set $A \cup B$ is $\{x : x \in A \vee x \in B\}$

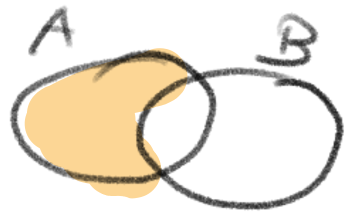


- intersection $A \cap B$

  the set $A \cap B$ is $\{x : x \in A \wedge x \in B\}$

- Difference $A \setminus B$ $(A - B)$

  the set $A \setminus B$ is $\{x : x \in A \wedge x \notin B\}$

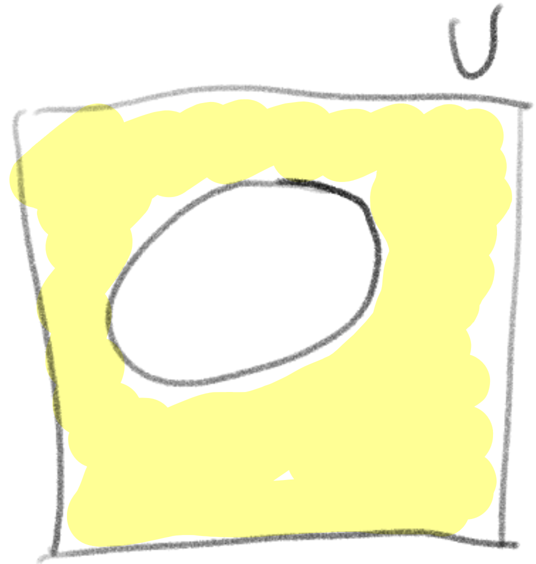


Complement of a set

complement of A : A^c

the SET $\{x : x \notin A\}$

$\phi^c = U$

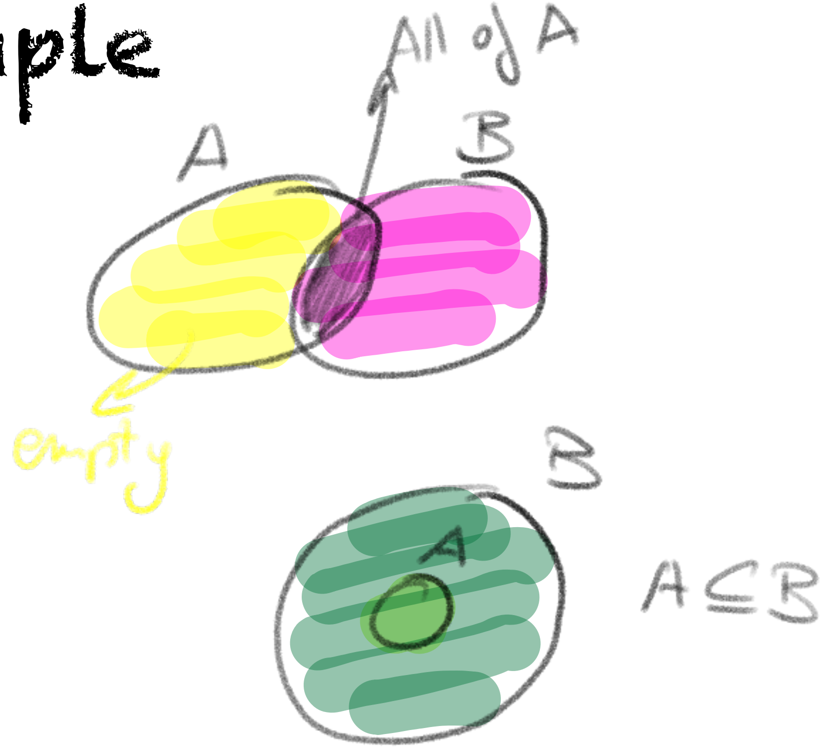


Proofs with sets: example

$$A \subseteq B \Leftrightarrow A \cup B = B$$

$\Rightarrow$ | $(A \subseteq B) \Rightarrow (A \cup B = B)$

$\Leftarrow$ | $(A \cup B = B) \Rightarrow A \subseteq B$



Venn diagrams are NOT A PROOF !!!

$\Rightarrow$ | 1. $B \subseteq A \cup B$

Let $x \in B$

then $x \in B \vee x \in A$

then $x \in (A \cup B)$

2. $A \cup B \subseteq B$

Let $x \in A \cup B$

then $x \in A \vee x \in B$

if $x \in A$, then $x \in B$ (since $A \subseteq B$)

if $x \in B$, then $x \in B$

$\Rightarrow (A \cup B \subseteq B) \wedge B \subseteq (A \cup B)$

$\Rightarrow A \cup B = B$

$\Leftarrow ((A \cup B) = B) \Rightarrow A \subseteq B$

Let $x \in A$

then $x \in A \vee x \in B$

then $x \in (A \cup B)$

then $x \in B$ (since $A \cup B = B$)

$\hookrightarrow A \subseteq B$

$(A \subseteq B) \Rightarrow (B^c \subseteq A^c)$

Proof 1. Let $x \in B^c$
then $x \notin B$ (def.)

then $x \notin A$
(if $x \in A$, then also $x \in B$, because $A \subseteq B$)

so $x \in A^c$

$\Rightarrow B^c \subseteq A^c$

Proof 2. $A \subseteq B$

$\Leftrightarrow \forall x : (x \in A) \to (x \in B)$ (def. of subset)

$\Leftrightarrow \forall x : (x \notin B) \to (x \notin A)$ (contrapositive)

$\Leftrightarrow \forall x : (x \in B^c) \to (x \in A^c)$ (def of complement)

$\Leftrightarrow B^c \subseteq A^c$ (def of subset)

Associativity

$$(1+2)+3 = 1+(2+3)$$

addition is associative

↳ it doesn't matter where I put the brackets

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

"intersection" is associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$

"union" is associative

Distributive Laws

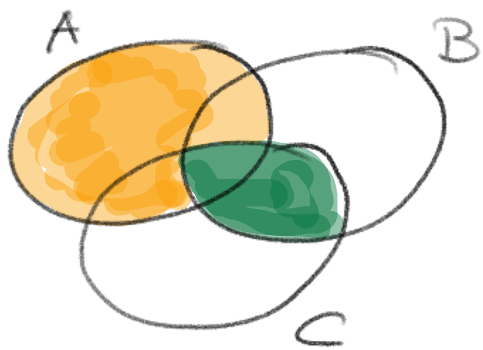
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$2 \times (3+4) = (2 \times 3) + (2 \times 4)$$

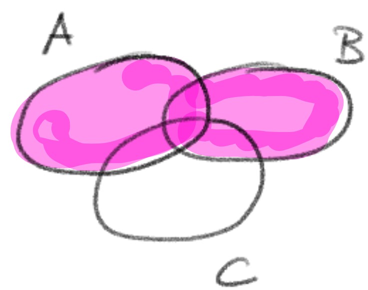
multiplication distributes over addition

Venn-diagram $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C)$



$A \cup (B \cap C)$ – everything that is coloured

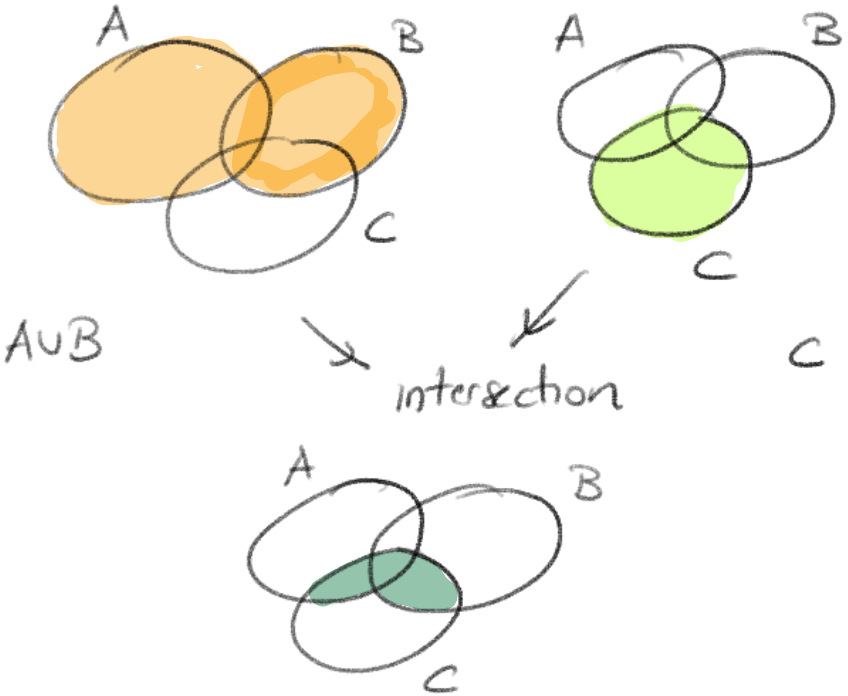


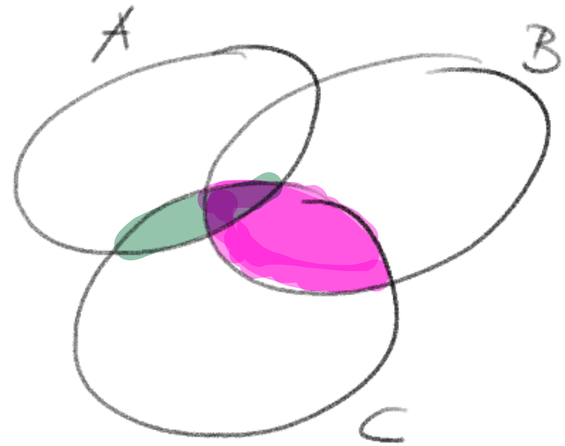
$A \cup B$

$A \cup C$

$(A \cup B) \cap (A \cup C)$ = the area coloured in the two plots

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$





union = everything that is coloured.

Proof: DIY

Remember: Venn diagrams are not a proof!

De Morgan laws

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

You can use the associative, distributive and De Morgan laws without a proof (but proving them is still a good exercise).

Checklist

- Do you understand how set membership works?
- Do you understand the definition of a subset, and how to prove that set A is a subset of B?
- Do you understand the meaning of intersection, union, complement, difference of sets?
- Do you know how to use Venn diagrams to help develop an intuition?
- Do you know how to prove that two sets are equal?