# Course overview

- Logic (week 1)
- Proof techniques (weeks 1–2)
- Set theory (weeks 2–3)
- Relations (weeks 3–4)
- Functions (week 4)
- Combinatorics (this week)

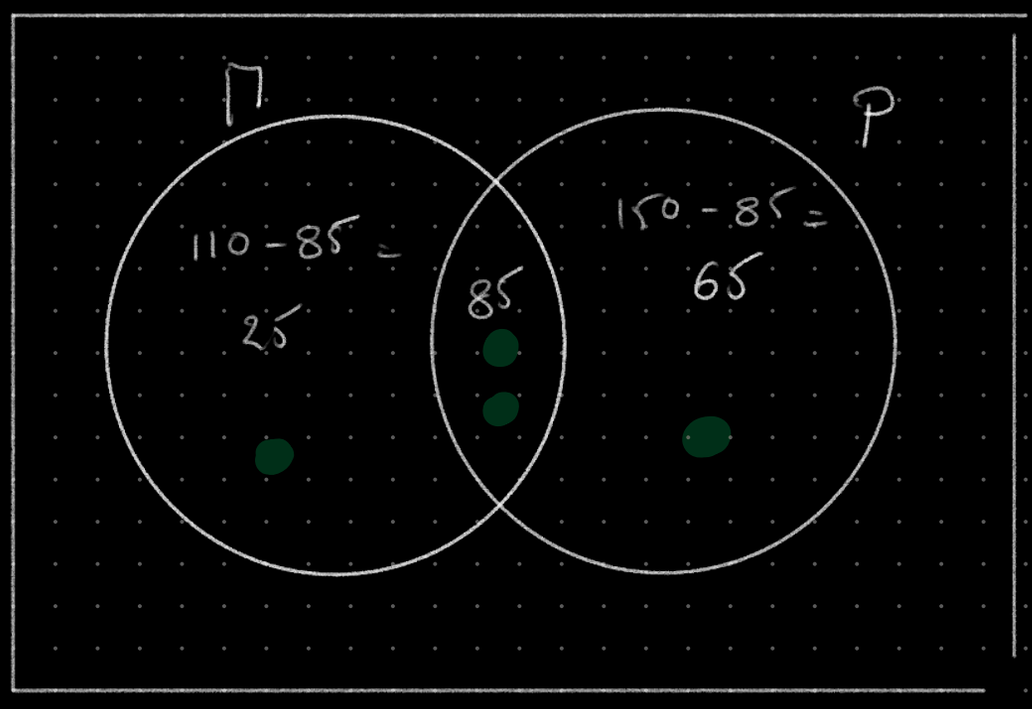
# Combinatorics (mathematics of counting)

- Inclusion-exclusion
- Rule of sum and rule of product
- Permutations and combinations
- Selections with/without order, with/without repetition

Today: Book section 2.5 and section 4.2

# Inclusion-exclusion

Example: From a total of 250 students, 110 students choose an elective course on Mathematics. 150 students are enrolled in a Physics course. 85 of those take both Physics and Mathematics. How many students don't take either course?



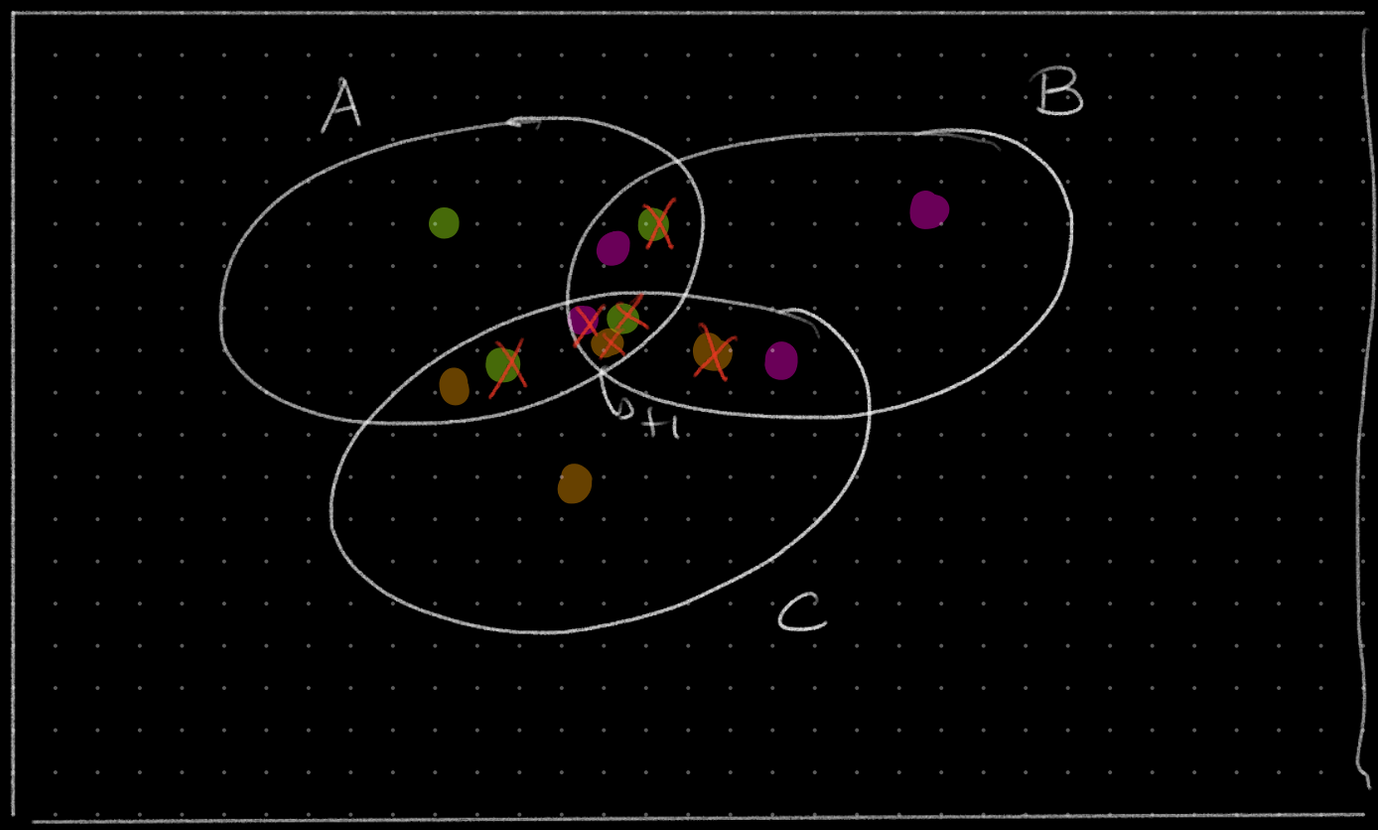

$|M| = 110$

$|P| = 150$

$|U| = 250$

$|U \setminus (P \cup M)| = 250 - 25 - 85 - 65 = 75$

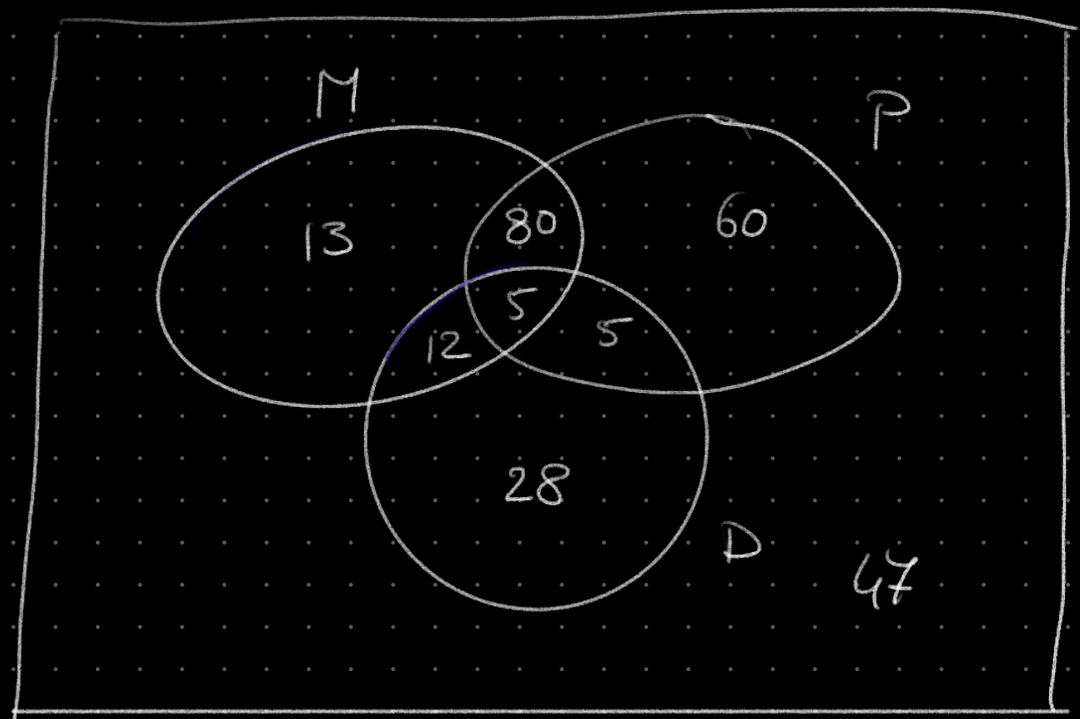
$|M \cup P| = |M| + |P| - |P \cap M|$ (inclusion - exclusion rule for two sets)




$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

↳ inclusion-exclusion rule for 3 sets.

From a total of 250 students, 110 students choose an elective course on Mathematics. 150 students are enrolled in a Physics course. 85 of those take both Physics and Mathematics.
From those 250 students, 50 are Dutch. From the Dutch students, 17 take (at least) Mathematics, 10 take (at least) Physics, 5 take both. How many of the students who take none of the courses are not Dutch?



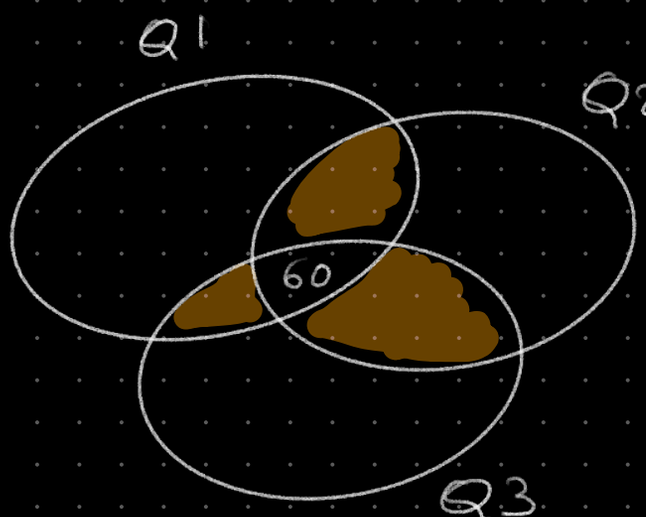

$$|U| - |M \cup P \cup D|$$

$$|M \cup P \cup D| = |M| + |P| + |D| - |M \cap P| - |M \cap D| - |P \cap D| + |P \cap D \cap M|$$

$$110 + 150 + 50 - (85 + 17 + 10) + 5$$

- 550 students do the Discrete Mathematics exam
- Every student could do at least 1 question
- 110 students could NOT do Q1 → 550 − 110 could do Q1 $|Q1| = 440$
- 165 students could NOT do Q2 $|Q2| = 550 - 165 = 385$
- 230 students could NOT do Q3 $|Q3| = 550 - 230 = 320$
- 60 students could do all 3 questions

How many students could do exactly 2 questions?




$$|Q1 \cup Q2 \cup Q3| = |Q1| + |Q2| + |Q3| - (|Q1 \cap Q2| + |Q2 \cap Q3| + |Q1 \cap Q3|) + |Q1 \cap Q2 \cap Q3|$$

what is asked

$|Q1 \cap Q2| - |Q1 \cap Q2 \cap Q3|$
$|Q1 \cap Q3| - |Q1 \cap Q2 \cap Q3|$
$|Q2 \cap Q3| - |Q1 \cap Q2 \cap Q3|$
$\times -180$

$$550 = 440 + 385 + 320 - x + 60$$

$$\Rightarrow -x = 550 - 440 - 385 - 320 - 60 = -655$$

$$x = 655$$

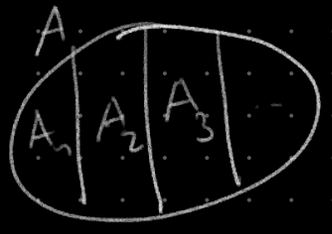
→ answer: $655 - 180 = 475$

# Combinatorics - sum and product

Rule of sum: If a set of objects can be divided into disjoint subsets, then the total number of objects is the sum of the number of objects in each subset.

CS : 297 students
DSAI : 240 students

There are $297 + 240$ options to choose a student representative.




$|A| = |A_1| + |A_2| + \dots$

Rule of product: when the objects we count have multiple parts, that can be selected independently, the total number of different objects is the product of the number of options for each part.

↳ there are 240 ways to choose a DSAI representative
there are 297 ways to choose a CS representative

⇒ there are $240 \times 297$ ways to choose a committee of a DSAI and a CS representative.

I am taking 2 pieces of luggage on a flight. I have 3 types of luggage:

- 3 suitcases
- 4 rucksacks
- 2 holdalls

How many ways can I select 2 different pieces of luggage?

$|S||R| + |R||H| + |S||H|$

$12 + 8 + 6 = 26$

(Note : if 2 pieces of the same type were allowed, the correct answer would be 36 → see next lecture)

Counting with / without order / repitition.

{A, B, C} → in how many ways can we select 2 letters?

- AA, AB, AC, BA, BB, BC, CA, CB, CC (9)
  with repetition. with order.

- repetition not allowed, order matters
  AB, AC, BA, BC, CA, CB (6 options)

- repetition not allowed, order does not matter
  AB, AC, BC (3 options)

- repetition allowed, order does not matter
  AA, AB, AC, BB, BC, CC (6 options)

- 6 friends eating icecream, 10 choices of icecream.
  - → order matters
  - → repetition allowed

- 6 persons, how many seating arrangements
  - → order important
  - → repetition not allowed

- 7 exam questions, you need to solve 4
  - → order not important
  - → repetition not allowed.

- 3 dice, how many different outcomes?
  repetition allowed, order not important

- How many strings of 8 bits can we make with exactly 3 ones?

  01001100
  11100000
  10101000

  → choose 3 positions out of 8
  repetition not allowed
  order does not matter

## Checklist

- Do you know the inclusion-exclusion formula for 2 and 3 sets?
- Can you use the inclusion-exclusion formula, and first principles to solve counting problems?
- Can you apply the sum and product rule to simple counting problems?

To be continued tomorrow