Discrete Mathematics

Introduction

Otti D'Huys

Organisation

- People
  - Course coordinator: Marieke Musegaas
  - Lecturers: Otti D'Huys (group 1), Marieke Musegaas (group 2)
  - Teaching Assistents: Martin Frohn, Ruben Meuwese, Martijn Elands, Marie Picquet, Adriana Purici, Maja Gojska, Tim Dick, Fivos Tzavellos, Iasonas Tsagkaris, Heinz Doss, Zirui Wang
- Lectures (group 1)
  - Tuesday, 8h30-10h30 (all weeks)
  - Wednesday 13h30-15h30 (weeks 1-5)
  - Only this week: Thursday 16h-18h
- Tutorials
  - Friday, 13h30-15h30

Organisation

- Material
  - Video clips
  - Textbook, ‘Discrete Mathematics’, A. Chetwynd and P. Diggle, Butterworth & Heinemann, available as e-book.
  - Lecture notes
  - Model solutions
- Assessment
  - Final written closed book exam, 100%
  - Optional practice exam week 5
- How to get help?
  - Feel free to ask any questions during the tutorials!
  - Discussion board
  - No emails!

Overview of the course

- Lecture 1: Propositional logic
- Lectures 2 - 4: Proofs
- Lectures 5 - 6: Set theory
- Lectures 7 - 9: Relations and functions
- Lectures 10-11: Combinatorics
- Lecture 12: revision

Discrete Mathematics

- ‘Discrete’ mathematics is the opposite of ‘continuous’ mathematics (Calculus,...) - while continuous mathematics mostly used to describe the physical world, discrete mathematics is the mathematical basis of computer science.
- The name of the course is poorly chosen: the topics of this course are fundamental to all branches of mathematics: logic, algebra, probability theory, calculus, and have applications in graph theory, cryptography, game theory,...
- Mathematics is the only language in which you can express yourself with absolute clarity (?) - this course teaches you how.

Lecture 1: Logic

- Numbers
- Divisibility
- Propositions
- Negation
- Conjunction
- Disjunction
- Truth tables
- Implication
- Biconditional

Book: Chapter 1, Sections 1.1, 1.2, 1.3

Integers

- Whole numbers
- $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
- Mathematical symbol: $\mathbb{Z}$

$$0 \underbrace{\in}_{\text{in / is an element}} \mathbb{Z}$$

Natural numbers

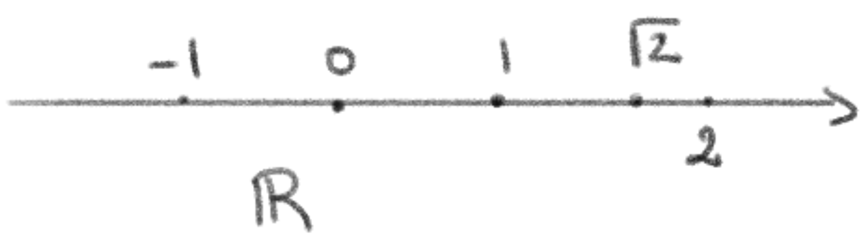
- Integers that are greater or equal to 1
- $1, 2, 3, \ldots$
- 0 is not a natural number (at least, not in this course)
- Mathematical symbol: $\mathbb{N}$

Rational numbers

- Any number that can be written as $\frac{a}{b}$, where a and b are both integers and $b \neq 0$.
- $1, 0.25, -0.333\ldots, \frac{9}{10}, 0, \ldots$
- π is not a rational number.
- Mathematical symbol: $\mathbb{Q}$

Real numbers

- Basically, this is the set of all numbers
- $0, -8.5, \pi, e, \sqrt{2}, 0.9, \ldots$
- A real number can be represented as a quantity along a continuous line ('the real line')
- Mathematical symbol: $\mathbb{R}$



Divisibility

- An integer m is divisible by an integer k if there is an integer n, such that $m = kn$

  example: 36 is divisible by 9, since $36 = 4 \cdot 9$

- Even numbers: an integer that is divisible by 2

  ↳ m is even, if there is a $n \in \mathbb{Z}$, such that $m = 2n$
- Odd numbers:

  an integer that is not even. $m = 2n + 1$
- Prime number: a natural number larger than 1, that can only be divided by 1 and itself.

Propositions

A propostion is a formal statement that is true or false (but not a matter of opinion).

- Examples of propositions
  - Maastricht is in Belgium
  - $1+1=2$
- Not a proposition:
  - How are you?
- Simple *primitive* or *atomic* propositions are the building blocks of *compound* propositions
  - Notation: p, q

Negation (not)

- We can negate a proposition:
  '37 is an odd number' $\rightarrow$ 37 is NOT an odd number
  p $\neg p$
- The truth value is opposite: the negation of a true proposition is false, the negation of a false proposition is true.
- Notation: $\neg p$, not p, $\sim p$, $\bar{p}$,
  Always be consistent in your notation!
- Double negation: $\neg(\neg p) = p$

Conjunction (and)

- **and** joins two propositions together.
- Notation: $p \wedge q$
- *p* **and** *q* is true if p is true and q is true
- Example:
  - 37 is a prime number **and** 7 is an even number. $\longrightarrow$ F

    T F
  - 5 is a prime number **and** 7 is an odd number. $\longrightarrow$ T

    T T

Disjunction (or)

- **or** joins two propositions together.
- Notation: $p \vee q$
- *p* **or** *q* is true if p is true, q is true, or both
- Example
  - 5 is a prime number **or** 7 is an even number. T
    T F
  - 5 is a prime number **or** 7 is an odd number. T
    T T
  - 9 is a prime number **or** 7 is an even number F
    F F

Brackets

When joining many propositions, you need brackets to disambiguate.
Example:

- p: 8 is a prime number. F
- q: 9 is divisible by 4. F
- r: $1 + 1 = 2$ T

- $(p \wedge q) \vee r$ $\rightarrow$ true

  F T

- $p \wedge (q \vee r)$ $\rightarrow$ false

  F T

Truth tables

- Systematic list of all the possible combinations of truth values of the atomic propositions + truth value of compound proposition
- number of rows: 2^N, with N the number of atomic propositions
- number of columns: as many as you need to keep the overview.

p	$\neg p$
0	1
1	0

p	q	$p \wedge q$	$p \vee q$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Truth tables: example

Using a truth table to evaluate a compound proposition:

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

Equivalent propositions

Two propositions are logically equivalent when they have the same truth table:

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Tautologies and contradictions

A tautology: a proposition that is always true.

Example: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
0	1	1
1	0	1

A contradiction: a proposition that is never true.

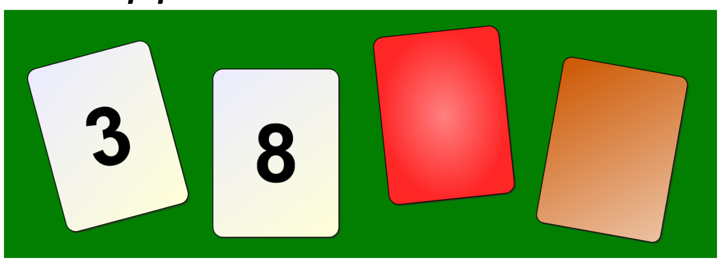
Example: $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
0	1	0
1	0	0

Wason's selection task

There are four cards, a simple rule. Which cards do you need to turn over to see if the rule has been broken?

If a card has an even number, then it's opposite face is red



The cards represents people's age, and their drink. Anyone younger than 18 is not allowed to consume alcohol.

Wason, P. C. (1968). "Reasoning about a rule". Quarterly Journal of Experimental Psychology. 20 (3): 273–281
https://en.wikipedia.org/wiki/Wason_selection_task
http://www.philosophyexperiments.com/wason/Default.aspx

Conditional propositions

- $p \to q$ $p \Rightarrow q$
- Read as: if p, then q p implies q
- Wason:
  - p: drinking alcohol
  - q: older than 18
  - $p \to q$: if you drink alcohol, you are older than 18
- If p is true, q is true
- If p is false, q can be anything

↳ $p \to q$ always true

$p \rightarrow q$

- if Maastricht is in Belgium, the moon is made of cheese. T

- if $1+1=3$, 7 is a prime number. T

Conditional propositions - equivalences

The following propositions are equivalent

- $p \to q$
- $\neg q \to \neg p$
- $\neg p \vee q$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$\neg p \vee q$
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	0	0	0
1	1	1	0	0	1	1

Conditional expressions

Which of the following inferences are valid? (i.e. if the premises above the line are true, it follows that the conclusion below the line is also true)

If I take my medication, I will get better.
I do not take my medication.

Therefore, I do not get better.

If I take my medication, I will get better.
I do not get better.

Therefore, I did not take my medication.

If I take my medication, I will get better.
I get better.

Therefore, I took my medication.

If I take my medication, I will get better.
I take my medication.

Therefore, I will get better.

The biconditional

- $p \leftrightarrow q$
- Short for: $(p \to q) \wedge (q \to p)$
- Read as: if and only if
- Example:

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

Checklist

1. Do you know the difference between natural numbers, integers, rational and real numbers?
2. Do you know what a prime number is?
3. Do you understand the meaning of, and how to use, the logical operators $\wedge$ (and), $\vee$ (or), $\neg$ (not)?
4. Do you feel comfortable understanding the meaning of brackets in a logical proposition?
5. Can you construct and fill in a truth table?
6. Do you understand how to show that two logical propositions are equivalent?
7. Do you understand the meaning of, and how to use, the logical operators $\rightarrow/\Rightarrow$ (if...then) and $\leftrightarrow/\Leftrightarrow$ (if and only if) ?