Conditional propositions

- $p \to q$
- Read as: If p, then q
  p implies q
  p is a sufficient condition for q
- If p is true, q must also be true
- If p is false, we have no information about q $p \to q$ true
- Examples:
  - If $1 + 1 = 3$, then the moon is made of cheese. True
  - If 9 is divisible by 3, 7 is not divisible by 3. True
  - If 9 is divisible by 3, all integers are even. False
  - If $1 + 1 = 3$, then $2 + 3 = 5$. True
- Note: $p \to q \neq q \to p$

Conditional propositions - equivalences

The following propositions are equivalent

- $p \to q$
- $\neg q \to \neg p$ contrapositive
- $\neg p \vee q$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	F	F	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

The biconditional

- $p \leftrightarrow q$
- Short for: $(p \rightarrow q) \wedge (q \rightarrow p)$
- Read as: p is equivalent to q
  p if and only if q
  p iff q

p is a necessary and sufficient condition for q

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T